Name ______________________ Class ____________ Date ________

2.2 Solving Absolute Value Equations

Essential Question: How can you solve an absolute value equation?

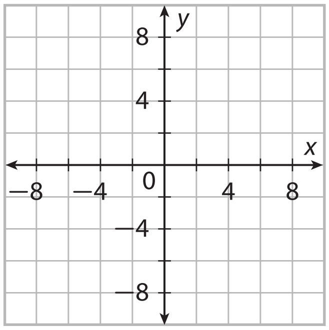
Resource Locker

Explore Solving Absolute Value Equations Graphically

Absolute value equations differ from linear equations in that they may have two solutions. This is indicated with a **disjunction**, a mathematical statement created by a connecting two other statements with the word "or." To see why there can be two solutions, you can solve an absolute value equation using graphs.

(A) Solve the equation $2|x - 5| - 4 = 2$.

Plot the function $f(x) = 2|x - 5| - 4$ on the grid. Then plot the function $g(x) = 2$ as a horizontal line on the same grid, and mark the points where the graphs intersect.



(B) Write the solution to this equation as a disjunction:

$x =$ ______ or $x =$ ______

Reflect

1. Why might you expect most absolute value equations to have two solutions? Why not three or four?

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2. Is it possible for an absolute value equation to have no solutions? one solution? If so, what would each look like graphically?

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Explain 1 Solving Absolute Value Equations Algebraically

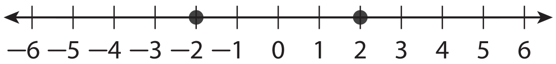
To solve absolute value equations algebraically, first isolate the absolute value expression on one side of the equation the same way you would isolate a variable. Then use the rule:

If $|x| = a$ (where a is a positive number), then $x = a$ OR $x = -a$.

Notice the use of a **disjunction** here in the rule for values of x. You cannot know from the original equation whether the expression inside the absolute value bars is positive or negative, so you must work through both possibilities to finish isolating x.

Example 1 **Solve each absolute value equation algebraically. Graph the solutions on a number line.**

(A) $|3x| + 2 = 8$



Subtract 2 from both sides. $|3x| = 6$

Rewrite as two equations. $3x = 6$ or $3x = -6$

Solve for x. $x = 2$ or $x = -2$

(B) $3|4x - 5| - 2 = 19$

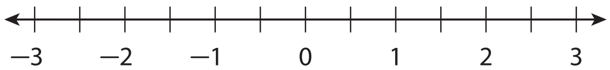
Add 2 to both sides. $3|4x - 5| = \boxed{\quad}$

Divide both sides by 3. $|4x - 5| = \boxed{\quad}$

Rewrite as two equations. $4x - 5 = \boxed{\quad}$ or $4x - 5 = \boxed{\quad}$

Add 5 to all four sides. $4x = \boxed{\quad}$ or $4x = \boxed{\quad}$

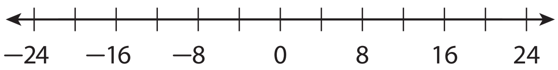
Solve for x. $x = \boxed{\quad}$ or $x = -\dfrac{\boxed{\quad}}{\boxed{\quad}}$



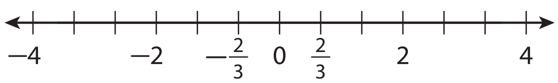
Your Turn

Solve each absolute value equation algebraically. Graph the solutions on a number line.

3. $\frac{1}{2}|x + 2| = 10$



4. $-2|3x - 6| + 5 = 1$



Explain 2 Absolute Value Equations with Fewer than Two Solutions

You have seen that absolute value equations have two solutions when the isolated absolute value expression is equal to a positive number. When the absolute value expression is equal to zero, there is a single solution because zero is its own opposite. When the absolute value is equal to a negative number, there is no solution because absolute value is never negative.

Example 2 **Isolate the absolute value expression in each equation to determine if the equation can be solved. If so, finish the solution. If not, write "no solution."**

(A) $-5|x + 1| + 2 = 12$

Subtract 2 from both sides.	$-5|x + 1| = 10$
Divide both sides by -5.	$|x + 1| = -2$
Absolute values are never negative.	No Solution

(B) $\frac{3}{5}|2x - 4| - 3 = -3$

Add 3 to both sides.	$\frac{3}{5}|2x - 4| = \boxed{}$
Multiply both sides by $\frac{5}{3}$.	$|2x - 4| = \boxed{}$
Rewrite as one equation.	$2x - 4 = \boxed{}$
Add 4 to both sides.	$2x = \boxed{}$
Divide both sides by 2.	$x = \boxed{}$

Your Turn

Isolate the absolute value expression in each equation to determine if the equation can be solved. If so, finish the solution. If not, write "no solution."

5. $3\left|\frac{1}{2}x + 5\right| + 7 = 5$

6. $9\left|\frac{4}{3}x - 2\right| + 7 = 7$

Elaborate

7. Why is important to solve both equations in the disjunction arising from an absolute value equation? Why not just pick one and solve it, knowing the solution for the variable will work when plugged backed into the equation?

8. **Discussion** Discuss how the range of the absolute value function differs from the range of a linear function. Graphically, how does this explain why a linear equation always has exactly one solution while an absolute value equation can have one, two, or no solutions?

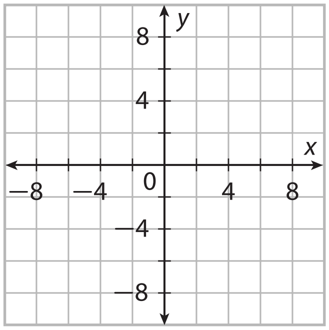
9. **Essential Question Check-In** Describe, in your own words, the basic steps to solving absolute value equations and how many solutions to expect.

Evaluate: Homework and Practice

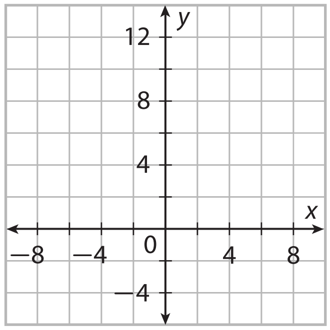
Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

Solve the following absolute value equations by graphing.

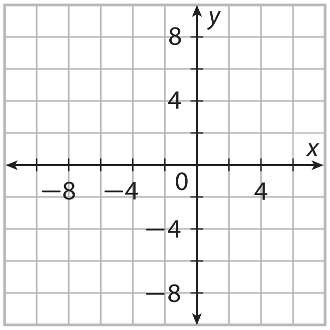
1. $|x - 3| + 2 = 5$



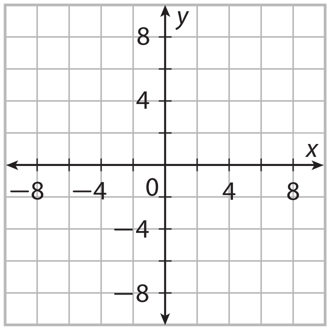
2. $2|x + 1| + 5 = 9$



3. $-2|x + 5| + 4 = 2$

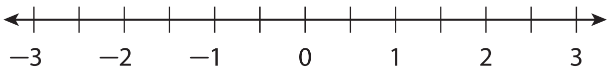


4. $\left|\frac{3}{2}(x - 2)\right| + 3 = 2$

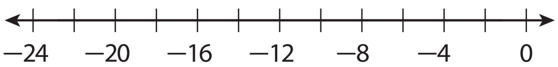


Solve each absolute value equation algebraically. Graph the solutions on a number line.

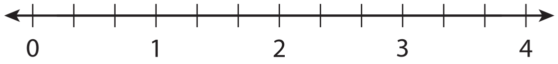
5. $|2x| = 3$



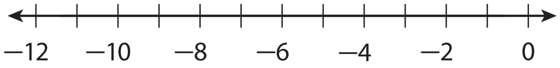
6. $\left|\frac{1}{3}x + 4\right| = 3$



7. $3|2x - 3| + 2 = 3$

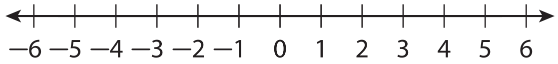


8. $-8|-x - 6| + 10 = 2$

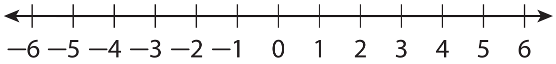


Isolate the absolute value expressions in the following equations to determine if they can be solved. If so, find and graph the solution(s). If not, write "no solution".

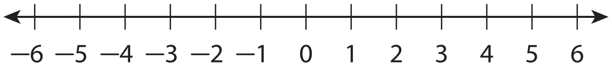
9. $\frac{1}{4}|x + 2| + 7 = 5$



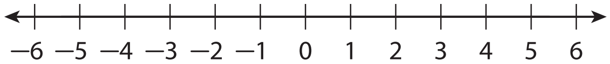
10. $-3|x - 3| + 3 = 6$



11. $2\left(|x + 4| + 3\right) = 6$

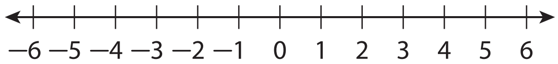


12. $5|2x + 4| - 3 = -3$

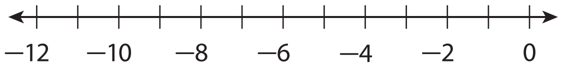


Solve the absolute value equations.

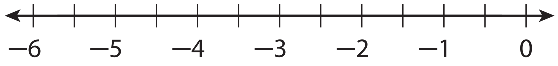
13. $|3x - 4| + 2 = 1$



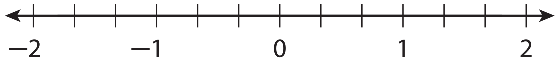
14. $7\left|\frac{1}{2}x + 3\frac{1}{2}\right| - 2 = 5$



15. $\left|2(x+5)-3\right|+2=6$

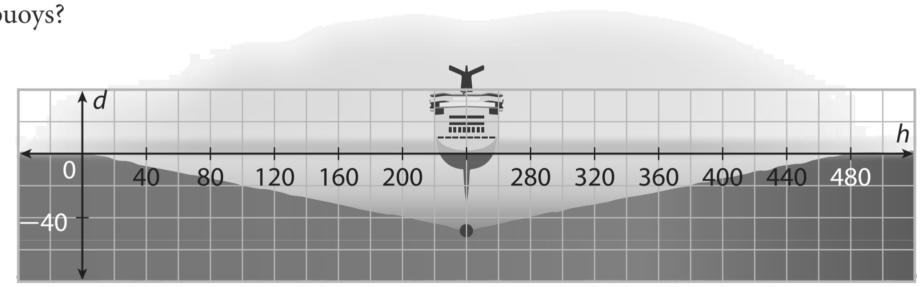


16. $-5\left|-3x+2\right|-2=-2$



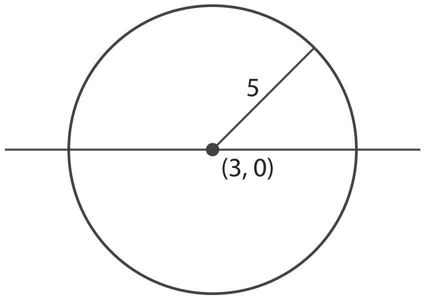
17. The bottom of a river makes a V-shape that can be modeled with the absolute value function, $d(h)=\frac{1}{5}\left|h-240\right|-48$, where d is the depth of the river bottom (in feet) and h is the horizontal distance to the left-hand shore (in feet).

A ship risks running aground if the bottom of its keel (its lowest point under the water) reaches down to the river bottom. Suppose you are the harbormaster and you want to place buoys where the river bottom is 30 feet below the surface. How far from the left-hand shore should you place the buoys?



18. A flock of geese is approaching a photographer, flying in formation. The photographer starts taking photographs when the lead goose is 300 feet horizontally from her, and continues taking photographs until it is 100 feet past. The flock is flying at a steady 30 feet per second. Write and solve an equation to find the times after the photographing begins that the lead goose is at a horizontal distance of 75 feet from the photographer.

19. **Geometry** Find the points where a circle centered at (3, 0) with a radius of 5 crosses the x-axis. Use an absolute value equation and the fact that all points on a circle are the same distance (the radius) from the center.



20. Select the value or values of x that satisfy the equation $-\frac{1}{2}|3x - 3| + 2 = 1$.

A. $x = \frac{5}{3}$ **B.** $x = -\frac{5}{3}$

C. $x = \frac{1}{3}$ **D.** $x = -\frac{1}{3}$

E. $x = 3$ **F.** $x = -3$

G. $x = 1$ **H.** $x = -1$

21. Terry is trying to place a satellite dish on the roof of his house at the recommended height of 30 feet. His house is 32 feet wide, and the height of the roof can be described by the function $h(x) = -\frac{3}{2}|x - 16| + 24$, where x is the distance along the width of the house. Where should Terry place the dish?

H.O.T. Focus on Higher Order Thinking

22. **Explain the Error** While attempting to solve the equation $-3|x - 4| - 4 = 3$, a student came up with the following results. Explain the error and find the correct solution:

$$-3|x - 4| - 4 = 3$$

$$-3|x - 4| = 7$$

$$|x - 4| = -\frac{7}{3}$$

$$x - 4 = -\frac{7}{3} \quad \text{or} \quad x - 4 = \frac{7}{3}$$

$$x = \frac{5}{3} \quad \text{or} \quad x = \frac{19}{3}$$

23. **Communicate Mathematical Ideas** Solve this absolute value equation and explain what algebraic properties make it possible to do so.

$3|x-2| = 5|x-2| - 7$

24. **Justify Your Reasoning** This absolute value equation has nested absolute values. Use your knowledge of solving absolute value equations to solve this equation. Justify the number of possible solutions.

$\left| |2x+5| - 3 \right| = 10$

25. **Check for Reasonableness** For what type of real-world quantities would the negative answer for an absolute value equation not make sense?

Lesson Performance Task

A snowball comes apart as a child throws it north, resulting in two halves traveling away from the child. The child is standing 12 feet south and 6 feet east of the school door, along an east-west wall. One fragment flies off to the northeast, moving 2 feet east for every 5 feet north of travel, and the other moves 2 feet west for every 5 feet north of travel. Write an absolute value function that describes the northward position, $n(e)$, of both fragments as a function of how far east of the school door they are. How far apart are the fragments when they strike the wall?